Computing Geometric and Mass Properties of Statues for Rocking Analysis

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ABSTRACT

The seismic response of statues is motivated by observations from recent earthquakes combined with their high cultural significance. Typically unrestrained with a high coefficient of static friction at its base, a statue can be analyzed as a rigid body which will undergo rocking during seismic excitation. A methodology is presented to obtain the critical geometric and mass properties required for numerical rigid body rocking analyses. Two nonintrusive approaches to capture the statue geometry are presented: light detection and ranging (LiDAR) and structure-from-motion (SfM), which generate a three-dimensional point cloud. A Poisson reconstruction algorithm intrinsically filters each point cloud and creates a fully enclosed triangulated surface mesh. Integrating over the surface mesh allows for computation of the required geometric and mass properties for numerical rocking analysis to simulate the seismic response of a statue.

PAST WORK

The analysis of cultural heritage artifacts has largely been a neglected area of earthquake engineering with advances only in the past few decades:

- Nigbor (1989) conducted a survey of statues within a museum collection and assessed their vulnerability through rigid body rocking analysis using envelope shapes and estimated mass properties.
- Due to the availability of laser scanners in recent years, higher accuracy models for statue analysis are more commonplace. Berto et al. (2012) presented a general preservation analysis of a set of statues in Florence, Italy, which included laser scans of statues, meshing, and solid element finite element analysis.

EQUIVALENT ROCKING ANALYSIS

Statues subject to seismic excitation can be expected to undergo a rocking-dominated response due to large aspect ratios (i.e. 3 < H/B < 5, where H = height to the center of mass and B = base dimension) and high coefficients of static friction ($\mu_s \approx 0.75$) . The equation of motion is:

$$\ddot{\theta} = -\frac{mRg}{I_{rp}}\left[\mathrm{sgn}(\theta)\sin(\alpha - |\theta|) + \frac{\ddot{u}_g}{g}\cos(\alpha - |\theta|)\right]$$

where θ is rotation, m is the mass, g is gravity, I_{rp} is the moment of inertia, and $\ddot{u}_g$ is ground acceleration. α and R are explained graphically in Figure 1.

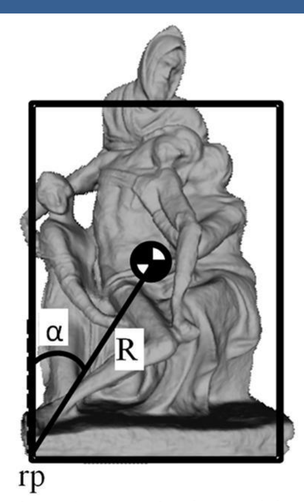

Fig. 1: Equivalent block over statue with relevant geometric parameters

DATA ACQUISITION

Light Detection and Ranging (LiDAR)

- Laser scanning is a commercially available, highly accurate method for recording geometry of structures and objects
- The scanner used for acquisition of the surveyed statues in this work is the FARO Focus 120 3D, a phase-based scanner (FARO 2011)
  - 3.8mm spot size
  - 0.009° step size
  - Close-range scans

  } potential for sub-millimeter resolution

Structure-from-Motion (SfM)

- SfM is 3D scene reconstruction using images, reconstructed using feature-tracking algorithms
- The statues surveyed were reconstructed from images using VisualSfM with corresponding dense reconstruction algorithm (Furukawa and Ponce 2010)
- The resultant unscaled point cloud can be scaled based on simple measurements (i.e. pedestal dimensions)

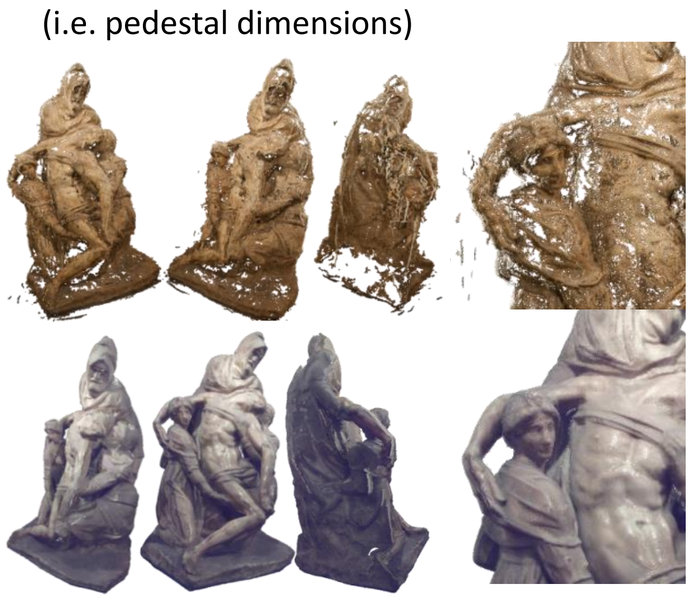

Fig. 2: Three isometric views and a detail of point clouds obtained by SfM (top) and LiDAR (bottom) of Michelangelo's *Florentine Pieta*.

- The SfM model consists of approx. 550k vertices; whereas, LiDAR has approx. 3.4 million vertices.
- Significant surface noise can be observed in the SfM model while LiDAR appears smoother.
- An area of occlusion is observed in the rear, top of the LiDAR model due to limitation s of scanner placement.
- A large area of occlusion is noticed in the SfM model on the entire rear of the statue due to its lack of detail (very few feature points to track).

MESHING & MASS PROPERTIES

The resultant point clouds developed from both SfM and LiDAR were filtered and triangulated using a Poisson surface reconstruction algorithm in Meshlab (Visual Computing Lab 2011). The meshes were resampled in order to recover a resolution via a clustering decimation algorithm. It is observed that final SfM meshes were 1-3% as dense as LiDAR meshes compared to the 10-15% difference pre-filtering.

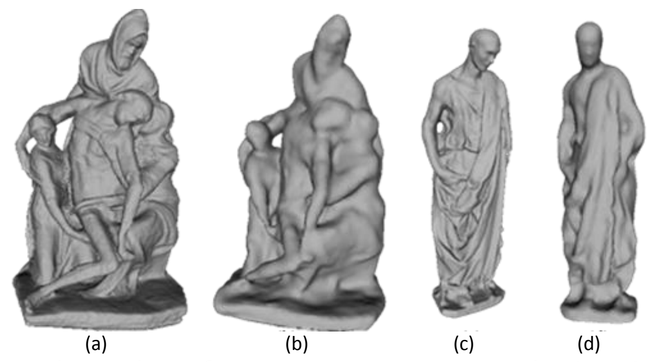

Fig. 3: Example meshes of two statues reconstructed using LiDAR, (a) and (c), and SfM, (b) and (d). Significantly more detail is maintained in LiDAR.

Many important properties important for rocking analysis require integration over the mass. The problem is reduced with an assumption of constant density and application of the divergence theorem resulting in integration over the surface (summation of triangles in mesh). Mass and centroid are calculated directly. Additional properties (see Equivalent Rocking Analysis) must be translated and/or calculated using the base dimensions, which are determined as follows: 1) determine edge vertices at base height, 2) calculate the mean of the edge vertices in the direction normal to plane or rocking (for rectangular footprint case). Refer to associated paper for discussion of cases of irregular footprint.

Table 1: Comparison of relevant geometric and mass properties for Florentine Pieta for varying resolutions of LiDAR, SfM, and envelope shapes.

		Envelope	SfM	LiDAR (10mm)	LiDAR (3mm)	LiDAR (1mm)
COM_z	[m]	1.1854	0.9675	0.9676	0.9211	0.9237
	[% diff]	*28.3%*	*4.7%*	*4.8%*	*-0.3%*	
I_{zz}	[kg·m^2]	887	343	387	387	388
	[% diff]	*128.6%*	*-11.6%*	*-0.3%*	*-0.3%*	
m	[kg]	5756	2801	2901	2902	2903
	[% diff]	*98.3%*	*-3.5%*	*-0.1%*	*0.0%*	
α_{xz}	[deg]	18.6	20.5	18.3	18.8	18.7
	[% diff]	*-0.5%*	*9.6%*	*-2.1%*	*0.5%*	
α_{yz}	[deg]	25.0	32.2	31.0	32.0	31.8
	[% diff]	*-21.4%*	*1.3%*	*-2.5%*	*0.6%*	

CONCLUSIONS & FUTURE WORK

- A methodology for determining geometric and mass properties of complex structures is presented
- Structure-from-Motion has the capability to capture the geometric and mass properties of complex structures (within 10% for the sample)
- Envelope shapes cannot reliably capture geometric and mass properties of complex structures (deviations greater than 100%)
- Mass and moment of inertia are the properties that are most affected by the resolution
- The degree to which the geometric and mass properties can vary with respect to rigid body analyses is an expected outcome of a current experimental campaign

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